SYNOPTIC:: Finite Deflection Discrete Element Analysis of Sandwich Plates and Cylindrical Shells with Laminated Faces, L. A. Schmit Jr. and G. R. Monforton, Case Western Reserve University, Cleveland, Ohio; *AIAA Journal*, Vol. 8. No. 8, pp. 1454–1461.

Structural Static Analysis; Structural Stability Analysis; Structural Composite Materials

Theme

Reports on a discrete element analysis capability for predicting the geometrically nonlinear response (including general instability) of sandwich plates and cylindrical shells with unbalanced laminated faces and orthotropic honeycomb cores. Calculated results for the finite deflection and elastic postbuckling behavior of sandwich and thin laminated systems are compared with analytical and experimental results given in the literature.

Content

The two faces of the sandwich system are considered to behave as thin shells and may be composed of an arbitrary number of bonded layers, each of which may have different thickness, linear elastic anisotropic material properties and orientation of elastic axes. The orthotropic core is considered relatively thick and representative of honeycomb sandwich cores. It is assumed that the transverse deflection is finite and uniform through the thickness of the sandwich.

Geometric nonlinearity is incorporated in the analysis by using the following set of finite-displacement, strain-displacement equations for the faces:

$$\epsilon_x = \partial u/\partial x + \tfrac{1}{2}(\partial w/\partial x)^2 - z\partial^2 w/\partial x^2 \qquad (1a)$$

$$\epsilon_y = \partial v/\partial y + w/R + \tfrac{1}{2}(\partial w/\partial y)^2 - z[\partial^2 w/\partial y^2 - (1/R)\partial v/\partial y] \qquad (1b)$$

$$\gamma_{xy} = \partial v/\partial x + \partial u/\partial y + (\partial w/\partial x)\partial w/\partial y - 2z[\partial^2 w/\partial x\partial y - (1/R)\partial v/\partial x] \qquad (1c)$$

where for cylindrical shells ($y = R\theta$) u and v are the membrane displacements in the longitudinal (x) and circumferential (θ) directions, respectively, and w is the transverse deflection. For plates, Eqs. 1 are valid by setting the curvature $1/R = 0$. The use of Eqs. (1), however, limits the validity of the formulation to cases where the rotations of the deformed geometry relative to the undeformed geometry are small.

Making use of Eqs. (1), the strain energy is expressed in terms of the geometry, the stiffnesses (membrane, coupling, and bending) of the faces, the core shear stiffnesses and the following displacement variables: the four membrane displacements u_f and v_f of the two faces ($f = 1,2$) and the transverse displacement w of the sandwich system. After adding the potential of the applied external loads to the strain energy, the discretized form of the potential energy for the discrete element is obtained by making use of assumed displacement functions.

The displacement functions used to approximate the displacement behavior are formed by the sum of products of one-dimensional first-order Hermite interpolation polynomials and undetermined nodal coefficients. The selection of the assumed displacement functions were based on geometric admissibility and completeness requirements as well as rigid body displacement considerations. The resulting rectangular plate and cylindrical shell elements incorporate a total of eighty degrees of freedom.

The formulation reported is such that transverse shear deformations in the core are accounted for and a wide range of boundary conditions is made available. For typical thin face sandwich systems, accurate stresses are predicted. The method is not bound to any specific micromechanics theory available for laminated structures, but the face considerations are bound to the Kirchhoff-Love assumptions. The analysis of thin anisotropic and transversely heterogeneous laminated plates and cylindrical shells can easily be accommodated as a specialization by considering only one face of the sandwich.

In order to illustrate the potential of the method for the finite deflection and elastic postbuckling analysis of sandwich and thin laminated structures, the following five numerical examples were studied: 1) finite deflection behavior of a clamped sandwich plate under uniform pressure; 2) postbuckling response of a compressed simply supported sandwich plate; 3) postbuckling behavior of a simply supported thin orthotropic plate; 4) postbuckling response of a simply supported thin unbalanced composite plate; 5) postbuckling behavior of a compressed sandwich cylindrical shell panel.

All solutions were obtained by searching for the local minimum of the discretized total potential energy using a scaled conjugate gradient algorithm. Where possible the results were evaluated and compared with existing analytical and/or experimental results and agreement was excellent.